

JJ In jj coupling, the interaction between l_1^* and s_1^* , and that between l_2^* and s_2^* are stronger than the interaction between l_1^* and l_2^* and that between s_1^* and s_2^* . Hence in this case ΔT_3 and ΔT_4 predominate over ΔT_1 and ΔT_2 in eq. (ii).

Since the angles between l_1^* and s_1^* , and between l_2^* and s_2^* are fixed, ΔT_3 and ΔT_4 are readily calculated from the cosine law :

$$\Delta T_3 = \frac{1}{2} a_3 (j_1^{*2} - l_1^{*2} - s_1^{*2}) \quad \dots \text{(xi)}$$

and
$$\Delta T_4 = \frac{1}{2} a_4 (j_2^{*2} - l_2^{*2} - s_2^{*2}), \quad \dots \text{(xii)}$$

But the angles between s_1^* and s_2^* , and between l_1^* and l_2^* are continually changing, and so the cosines in ΔT_1 and ΔT_2 must be averaged. The average values are given by

$$\overline{\cos (s_1^* s_2^*)} = \cos (s_1^* j_1^*) \cos (j_1^* j_2^*) \cos (j_2^* s_2^*)$$

and
$$\overline{\cos (l_1^* l_2^*)} = \cos (l_1^* j_1^*) \cos (j_1^* j_2^*) \cos (j_2^* l_2^*).$$

Using these average values in eq. (ii), we get

$$\begin{aligned} \Delta T_1 + \Delta T_2 &= [a_1 s_1^* s_2^* \cos(s_1^* j_1^*) \cos(j_2^* s_2^*) \\ &\quad + a_2 l_1^* l_2^* \cos(l_1^* j_1^*) \cos(j_2^* l_2^*)] \cos(j_1^* j_2^*) \\ &= \frac{1}{2} (a_1 \beta_1 + a_2 \beta_2) (J^{*2} - j_1^{*2} - j_2^{*2}), \quad \dots(\text{xiii}) \end{aligned}$$

where $\beta_1 = \frac{s_1^{*2} + j_1^{*2} - l_1^{*2}}{2 j_1^{*2}} \frac{s_2^{*2} + j_2^{*2} - l_2^{*2}}{2 j_2^{*2}} \quad \dots (\text{xiv})$

and $\beta_2 = \frac{l_1^{*2} + j_1^{*2} - s_1^{*2}}{2 j_1^{*2}} \frac{l_2^{*2} + j_2^{*2} - s_2^{*2}}{2 j_2^{*2}} \quad (\text{xv})$

Writing $a_1 \beta_1 + a_2 \beta_2 = B, \quad (\text{xvi})$

eq. (xiii) becomes

$$\Delta T_1 + \Delta T_2 = \frac{1}{2} B (J^{*2} - j_1^{*2} - j_2^{*2}). \quad \dots(\text{xvii})$$

We may again write any fine-structure term by the formula

$$T = T_0 - \Delta T_1 - \Delta T_2 - \Delta T_3 - \Delta T_4$$

where T_0 represents the hypothetical centre.

A good example of a ps configuration having jj coupling is found in tin. For ps configuration, we have

$$\begin{aligned} l_1 &= 1, s_1 = \frac{1}{2}; \quad \therefore j_1 = \frac{1}{2}, \frac{3}{2} \\ l_2 &= 0, s_2 = \frac{1}{2}; \quad \therefore j_2 = \frac{1}{2}. \end{aligned}$$

This gives rise to two (j_1, j_2) combinations, namely $(\frac{1}{2}, \frac{1}{2})$ and $(\frac{3}{2}, \frac{1}{2})$. We know that

$$J = |j_1 - j_2|, |j_1 - j_2| + 1, \dots, (j_1 + j_2).$$

Thus the J -values corresponding to the above combinations are 0, 1 and 1, 2 respectively. Thus we have the terms

$$\left(\frac{1}{2}, \frac{1}{2}\right)_{0,1} \quad \text{and} \quad \left(\frac{3}{2}, \frac{1}{2}\right)_{1,2}$$

The shift of each term from the centre of gravity is $\Delta T_3 + \Delta T_4$. Now,

$$\begin{aligned} \Delta T_3 + \Delta T_4 &= \frac{1}{2} a_3 (j_1^{*2} - l_1^{*2} - s_1^{*2}) \\ &\quad + \frac{1}{2} a_3 (j_2^{*2} - l_2^{*2} - s_2^{*2}). \end{aligned}$$

Putting $j_1 = \frac{1}{2}, j_2 = \frac{1}{2}$ and the values of l_1, s_1, l_2, s_2 , we get

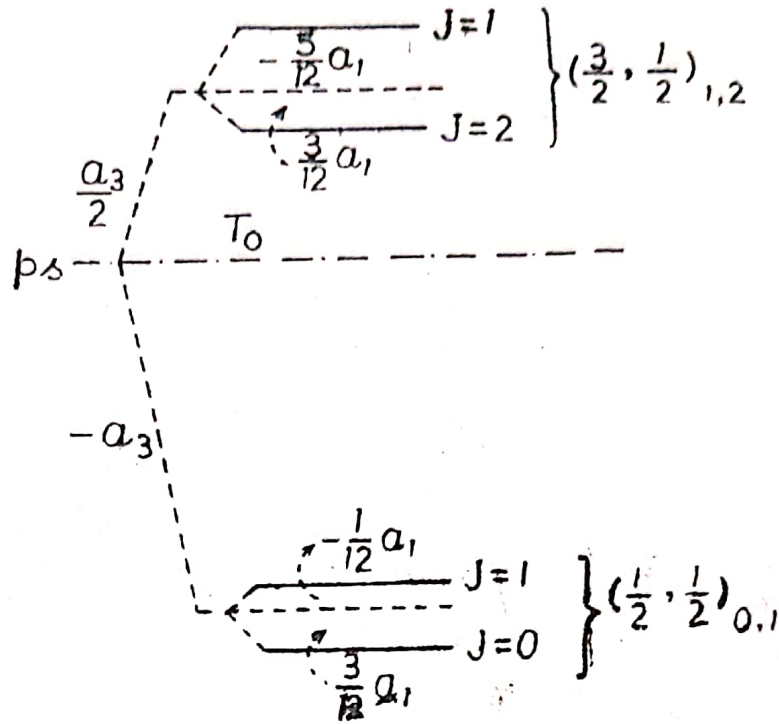
$$\Delta T_3 + \Delta T_4 = -a_3.$$

Thus the term $\left(\frac{1}{2}, \frac{1}{2}\right)_{0,1}$ is shifted down the hypothetical centre by a_3 .

Again, putting $j_1 = \frac{3}{2}, j_2 = \frac{1}{2}$ and the same other values, we get

$$\Delta T_3 + \Delta T_4 = \frac{a_3}{2}.$$

Thus the term $\left(\frac{3}{2}, \frac{1}{2}\right)_{1,2}$ is shifted up by $a_3/2$. This has been shown in Fig. 8.



(Fig. 8)

For the separation of fine-structure levels, we calculate $\Delta T_1 + \Delta T_2$, which is

$$\Delta T_1 + \Delta T_2 = \frac{1}{2} B (J^{*2} - j_1^{*2} - j_2^{*2}).$$

For the term $\left(\frac{3}{2}, \frac{1}{2}\right)_{1,2}$, we have

$$B = \frac{1}{3} a_1$$

$$\begin{aligned} \text{and } \Delta T_1 + \Delta T_2 &= -\frac{5}{4} B, \frac{3}{4} B \\ &= -\frac{5}{12} a_1, \frac{3}{12} a_1. \end{aligned}$$

For the term $\left(\frac{1}{2}, \frac{1}{2}\right)_{0,1}$ we have

$$B = -\frac{1}{3} a_1$$

$$\begin{aligned} \text{and } \Delta T_1 + \Delta T_2 &= -\frac{3}{4} B, \frac{1}{4} B \\ &= \frac{3}{12} a_1, -\frac{1}{12} a_1. \end{aligned}$$

These separations are also shown in Fig. 8. It can be seen in Fig. 8 that the total 3P separation, *i.e.* the interval between $J=0$ level and $J=2$ level is $3a_2/2$, which is the same as in $L-S$ coupling.